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Quantum engagement and microscopic quantum interferences *en masse* in a coherently mixed Bose gas

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Abstract

A recently proposed thought experiment involving two differently polarized boson gases (from two independent sources) coming together in a mixing chamber is revisited from a broader point of view. Coherent mixing in the chamber brings about a novel quantum statistical correlation. This correlation stems from the near-indistinguishability of the bosons in *spin* space, and manifests itself in the form of numerous microscopic quantum interferences occurring together in a macroscopic scale. It also gives rise to 'quantum engagement' of the two new spin eigenstates of the gas mixture. A parallel of the quantum engagement with the well known quantum entanglement between particles is brought out and explained.

1. Introduction

The Pauli exclusion principle gives rise to an effective two-body correlation that leads to the famous Heisenberg exchange interaction [1] and is largely responsible for magnetism. Similarly boson statistics harbors characteristic correlations that lead to, among other things, the Bose–Einstein condensation (BEC) [2–4]. As was first pointed out by Dirac [5], the correlations exhibited in both statistics owe their origins to the *extrinsic* indistinguishability that results from spatially overlapping wavepackets of otherwise identical particles in quantum mechanics. In fact, the various sizes and shapes of the wavepackets due to the overall confining potential or potential pockets [6], or due to the finite lifetimes of the bosons themselves [7] could play an important role in many situations.

In the following it will be shown that there exists a different type of correlation again of purely quantum statistical origin. The difference is manifested in two counts. Firstly, it only exists for *bosons* with nonvanishing overlap in *spin* space, *i.e.,* the space of *internal* states; hence it is ascribed to *intrinsic* indistinguishability [8, 11] rather than to extrinsic indistinguishability. Secondly, it is a correlation akin to quantum entanglement that arises from coherence [12–14]. This correlation was first alluded to recently in connection with BEC [8]. Upon probing into the deeper details we now find that this new correlation goes beyond BEC. It is also a manifestation of microscopic quantum interference but multiplied many folds, i.e., quantum interferences occurring *en masse* or together in a macroscopic scale. This is attributed to the bosonic statistics of the atoms involved.

2. A simplified gedanken experiment and the tuning parameter for indistinguishability

In a previous work [8] we started by asking the question how the degree of indistinguishability can be quantitatively defined and how it would affect quantum statistical thermodynamics such as the Bose–Einstein condensation (BEC). An idealized thought experiment involving a coherently mixed gas of bosons of two different polarizations was proposed. After an analysis by the method of density matrix, two chemical potentials corresponding to two distinct species in the mixed gas were found. They thus lead to two stages of BEC, each with its own predicted condensation temperature T_c. However, in order to maintain the integrity of the spin polarizations in spite of the spin–spin interactions a rather complex arrangement of several Stern–Gerlach set-ups was employed. In the present work we aim at a deeper study of the new quantum statistical correlation itself. To avoid unnecessary complications we are proposing a new experimental arrangement which is conceptually simplified and hence easier to visualize, with the integrity of the boson spins now taken for granted.

Consider a dilute gas of free bosonic atoms such as rubidium Rb^{87} in their ground state [15, 16]. A beam of these atoms from source A is polarized in the spin state $|F = 1, m_F = 1\rangle$ (abbreviated as $|1, 1\rangle$) by external means such as a uniform magnetic field along the z-axis (or a suitable Stern–Gerlach set-up), and another beam of the same atoms but from a different source B is polarized in $|F = 1, m'_F = 1\rangle$ (abbreviated as $|1, 1'\rangle$) by another magnetic field along the z'-direction that is tilted at an angle θ with the z-axis in the xz-plane.

To relate $|1, 1'\rangle$ to $|1, 1\rangle$ some elementary angular momentum algebra is involved. Exploiting the fact that an angular momentum or a spin state of $F = 1$ behaves like a spherical tensor of rank 1 and hence like an ordinary vector, we start with the normalized state $|\alpha\rangle$ defined by

$$b|\alpha\rangle \equiv \sin^2\frac{\theta}{2}|1,-1\rangle + \frac{\sin\theta}{\sqrt{2}}|1,0\rangle \tag{1}$$

and obtain eventually (with $a^2 + b^2 = 1$)

$$|1,1'\rangle = a|1,1\rangle + b|\alpha\rangle. \tag{2}$$

With $\langle 1, 1|\alpha\rangle = 0$, and $\langle\alpha|\alpha\rangle = 1$, the amplitudes a and b are determined to be

$$a \equiv \langle 1,1'|1,1\rangle = \cos^2\frac{\theta}{2} \tag{3}$$

and

$$b = \sqrt{\sin^2\frac{\theta}{2}\left(1+\cos^2\frac{\theta}{2}\right)}. \tag{4}$$

We now send the two polarized beams from the independent sources A and B into a common mixing chamber which is *free* of any external magnetic field. This arrangement is much easier to visualize than the one in [8]. The integrity of all the unprimed and primed states $|F = 1, m_F = 0, \pm 1$ or $m'_F = 0, \pm 1\rangle$ are assumed justifiable by referring back to our detailed discussions for the previous arrangement [8–10].

When the tilting angle $\theta = 0$, all the atoms in this chamber would be in the identical internal state $|1, 1\rangle$ and hence indistinguishable intrinsically. When θ deviates from 0 the atoms in $|1, 1\rangle$ would begin to differ from those in $|1.1'\rangle$ for $\langle 1, 1|1, 1'\rangle = \cos^2\frac{\theta}{2} = a \geqslant 0$. This overlap $\cos^2\frac{\theta}{2}$ of the two states in spin space thus parametrizes precisely the degree of *intrinsic indistinguishability* [8]. For a given θ it is a number of known magnitude and phase. In this sense we may say θ itself serves as a tuning parameter for the distinguishability.

3. What happens in the mixing chamber?

For an atom in the uniformly mixed gas, the single-particle orbitals are of two kinds, namely, $|l\rangle = |\vec{\boldsymbol{k}}; 1, 1\rangle$ and $|l'\rangle = |\vec{\boldsymbol{k}}; 1, 1'\rangle$ where $\vec{\boldsymbol{k}}$ denotes the translational freedom of the atom. Being from two independent sources each atom in the common chamber is in a mixed state. Exploiting the independence of the single-particle orbitals of the free gas, we can treat one orbital at a time. Furthermore, with the decoupling of the total atomic spin F from the translational freedom $\vec{\boldsymbol{k}}$ we may factorize the orbital state into the translational part and the spin part, $|l\rangle = |\vec{\boldsymbol{k}}\rangle|\phi_{\text{spin}}\rangle$. Here

$$|\phi_{\text{spin}}\rangle = \sqrt{P_1}\mathrm{e}^{\mathrm{i}\alpha}|1, 1\rangle + \sqrt{P_1'}\mathrm{e}^{\mathrm{i}\alpha'}|1, 1'\rangle \tag{5}$$

is not a pure state but rather a mixed state. It is the spin state of every single atom in the uniformly mixed gas. $P_1 = \frac{N_1}{N_1+N_1'}$ and $P_1' = \frac{N_1'}{N_1+N_1'}$ denote respectively the probabilities of the occupation of the states $|1, 1\rangle$ and $|1, 1'\rangle$ for any given associated state $|\vec{\boldsymbol{k}}\rangle$ provided, of course, this $|\vec{\boldsymbol{k}}\rangle$ is occupied, i.e., $n_{\vec{k},\mathbf{1}} \geqslant 1$, $n_{\vec{k},\mathbf{1}'} \geqslant 1$. Otherwise P_1 and P_1' would lose their meaning if $n_{\vec{k},\mathbf{1}} = 0$ or $n_{\vec{k},1'} = 0$. N_1 and N_1' are the number of atoms in the respective states $|1, 1\rangle$ and $|1, 1'\rangle$ originally from the independent sources A and B. Hence the relative phase α–α' is random in equation (5), guaranteeing $\langle\phi_{\text{spin}}|\phi_{\text{spin}}\rangle_{\text{av}} = 1$. It is important to note that equation (5) is valid only after the gases have been thoroughly and uniformly mixed so that the probabilities P_1 and P_1' in the single-particle state equation (5) make physical sense.

To investigate the statistical properties of the mixed gas the proper and the most convenient way is to employ the density matrix.

4. Treatment by density matrix

Let ρ_l be the density matrix for the orbital l which can correspondingly be factorized into $\rho_l = \rho_{\text{spin}}\rho_{\vec{k}}$. Here $\rho_{\vec{k}}$ describes only the translational motion, and ρ_{spin} is given by

$$\rho_{\text{spin}} = \{|\phi_{\text{spin}}\rangle\langle\phi_{\text{spin}}|\}_{\text{av}} = P_1|1, 1\rangle\langle 1, 1| + P_1'|1, 1'\rangle\langle 1, 1'| \tag{6}$$

after averaging over the random phases of $(\alpha$–$\alpha')$.

In the orthonormal basis of $|1, 1\rangle$ and $|\alpha\rangle$ of equation (2) the above ρ_{spin} becomes

$$\rho_{\text{spin}} = \rho_0 + \rho_{\text{coh}} \tag{7}$$

where

$$\rho_0 = \begin{pmatrix} P_1 + P_1'|a|^2 & 0 \\ 0 & P_1'|b|^2 \end{pmatrix} \tag{8}$$

and

$$\rho_{\text{coh}} = \begin{pmatrix} 0 & P_1'b^*a \\ P_1'ba^* & 0 \end{pmatrix}. \tag{9}$$

If $|1, 1'\rangle$ of equation (2) were not a pure state but rather a mixed state with random phase relation between the amplitudes a in $|1, 1\rangle$ and b in $|\alpha\rangle$, the two off-diagonal matrix elements in equation (9) would each become zero upon phase averaging. Thus ρ_{coh} is originated purely from coherence. In passing we should mention that while we have purposely made equation (9) formally identical to a corresponding equation in [8], the physical content of all the matrix elements are rather different; they refer to a different set of basis states and pertain to two different experiments.

It is now straightforward to diagonalize the above ρ_{spin} by

$$|\psi_\pm\rangle = A_\pm|1, 1\rangle + B_\pm|\alpha\rangle \tag{10}$$

whose eigenvalues are denoted by $P_\pm$. We find

$$P_\pm = \tfrac{1}{2}\left[1 \pm \sqrt{1 - 4P_1 P_1'|b|^2}\right] \tag{11}$$

and the coefficients $A_\pm$ and $B_\pm$ are given by

$$\frac{P_\pm - P_1 - P_1'|a|^2}{P_1' a b^*} = \frac{B_\pm}{A_\pm} = \frac{P_1' a^* b}{P_\pm - P_1'|b|^2} \tag{12}$$

where a and b are given by equations (3), (4).

With $\langle\psi_\pm|\psi_\pm\rangle = 1$ and $\langle\psi_+|\psi_-\rangle = 0$ the above $|\psi_+\rangle$ and $|\psi_-\rangle$ constitute a new set of orthonormal single-particle states. Together with the associated probabilities P_+ and P_- they are to be used to calculate the corresponding chemical potentials and condensation temperatures μ_+, T_+ and μ_-, T_- [8], as well as the value of any physical single-particle observable ϑ according to $\langle\vartheta\rangle = P_+\langle\psi_+|\vartheta|\psi_+\rangle + P_-\langle\psi_-|\vartheta|\psi_-\rangle$. Once the atoms get mixed uniformly inside the mixing chamber, ρ_{coh} of equation (9) becomes operative and this new set of $|\psi_\pm\rangle$ states will take over the old set of $|F = 1, m_F = 0, \pm 1$ or $m_F' = 0, \pm 1\rangle$ for any affair of the noninteracting atoms in that chamber. The old set has simply become obsolete.

In the limit of $\theta \ll \frac{\pi}{2}$, we find

$$P_+ \simeq 1 - \frac{9}{16} P_1 P_1' \theta^2, \qquad \frac{B_+}{A_+} \simeq \frac{3}{4} P_1' \theta; \tag{13}$$

and

$$P_- \simeq \frac{9}{16} P_1 P_1' \theta^2, \qquad \frac{B_-}{A_-} \simeq -\frac{4}{3P_1'\theta}. \tag{14}$$

We may generalize to the case of $M \geqslant 2$ beams of bosons coming into the mixing chamber. For example, instead of equation (6) we may have

$$\rho_{\mathrm{spin}} = P_1|1,1\rangle\langle 1,1| + P_1'|1,1'\rangle\langle 1,1'| + P_1''|1,1''\rangle\langle 1,1''| + \cdots. \tag{15}$$

It can be diagonalized as before by a corresponding set of two eigenstates with eigenvalues $P_\pm = \frac{1}{2}\{1 \pm \sqrt{1 - 4\sum_{i<j}^{M} P_i P_j |a_i b_j - a_j b_i|^2}\}$ which can be easily verified for the case of $M = 3$.

Back to our case of $M = 2$: we first observe that, with $P_1 = \frac{N_1}{N_1 + N_1'}$, $P_1' = \frac{N_1'}{N_1 + N_1'}$, it is only the ratio of the two populations $\frac{N_1'}{N_1}$ rather than the individual N_1 and N_1' that enters the constitution of the new states $|\psi_\pm\rangle$ of equations (10) and (12) as well as their associated $P_\pm$ of equation (11). This is highly significant. It means that they would remain unchanged when we lower the two densities $n_1 = \frac{N_1}{V}$ and $n_1' = \frac{N_1'}{V}$, as long as their ratio is kept fixed. While the corresponding lowered densities $n_\pm = P_\pm(n_1 + n_1')$ would affect the respective chemical potentials $\mu_\pm$ and hence depress the critical temperatures $T_\pm$ of the two distinct species of bosons [8], their states $|\psi_\pm\rangle$ would remain intact. As a bonus, the lowered densities not only help reduce the inter-particle interactions but also lessen the requirement for low temperatures, for wavepacket overlaps are no longer essential as far as the observable effects of $|\psi_\pm\rangle$ are concerned.

Now that BEC is not our main concern we could decrease the densities such that the wavepackets enveloping the atoms in general have little overlap with one another. The more familiar quantum statistical correlation caused by the previously discussed *extrinsic* indistinguishability [5] would then be lost. Yet there remains a seemingly mysterious correlation between the states $|\psi_+\rangle$ and $|\psi_-\rangle$.

5. Quantum engagement, a new type of quantum statistical correlation

Suppose we have N widely separated, noninteracting hydrogen atoms, some of them, say, N_1 in $|1s\rangle$, some others, say N_2 in $|2p_0\rangle$. Under ordinary circumstances their atomic states would not be altered into some other states like $|2p_1\rangle$, $|3d_{-2}\rangle, \ldots$ or some linear combinations thereof when a few more of the same original kinds, say ΔN_1 and ΔN_2, are introduced into their neighborhood. This does not seem to be the case when the atoms are in our $|\psi_+\rangle$ and $|\psi_-\rangle$.

In the well known example of quantum entanglement [14], a molecule containing two spin $\frac{1}{2}$-atoms in a singlet spin state of $S = 0$ is disintegrated. Yet, even when the two atoms have gone far apart in opposite directions from each other the two atomic spins remain correlated: when one of them is measured to be in $|\uparrow\rangle$ along a certain γ-direction, the measurement of the other spin far away will always yield $|\downarrow\rangle$ with respect to that same γ-direction, regardless of how we manipulate or tinker with this γ-direction. It is thus said that the two atomic spins are entangled [12–14], despite the absence of interaction between the two separated atoms. In our present case, the atomic state $|\phi_{\rm spin}\rangle$ of equation (5) in the two differently polarized gases before mixing coherently as they are just entering the mixing chamber can be likened to the entangled $S = 0$ state of the molecule *before* disintegration. No special correlation is identifiable yet. Once well settled inside that chamber, coherent interference occur and the new set of $|\psi_\pm\rangle$ states takes over, much like the two atomic spin states of the *disintegrated* molecule. A correlation akin to entanglement manifests itself in the following way. When one of the eigenstates $|\psi_+\rangle$ within the chamber is varied, say, by changing n_1 and hence P_1, (keeping in mind that $P_1' = 1 - P_1$) the other state $|\psi_-\rangle$ within the chamber will be induced to change according to equation (12), and vice versa. The varying of $|\psi_+\rangle$ by tinkering with n_1 can be likened to the act of measuring one of the two widely separated atomic spins as we tinker with the direction γ. An entanglement-like act ensues as the other member $|\psi_-\rangle$ that can be likened to the other atomic spin in molecular disintegration is induced to change accordingly.

Yet there is more to it. For there may already be n_+ atoms in $|\psi_+\rangle$ and n_- of them in $|\psi_-\rangle$. Once n_1, say, is changed by Δn_1, not just one but *every* atom originally in $|\psi_+\rangle$ and in $|\psi_-\rangle$ will suffer the same change of state, $\Delta|\psi_+\rangle$ or $\Delta|\psi_-\rangle$. This is, therefore, a new type of correlation not encountered before. We shall call this correlation by the name of 'quantum engagement'; it is between the $|\psi_+\rangle$ and $|\psi_-\rangle$ states themselves rather than between a certain pair of atoms which happens to be in them. It depends crucially not only on the intrinsic indistinguishability [8] but also on the variability of the parameter n_1/n_1' through which this engagement makes its appearance. Therefore, it is fundamentally quantum statistical in nature. In contrast, quantum entanglement, for example, in the example of molecular disintegration, is a purely quantum mechanical correlation between the two atom spins in the product of the disintegration. These two atoms do not even have to be of the same kind. Changing the number of other similar molecules in its neighborhood would not affect the outcome of entanglement manifested by our disintegrated molecule at hand.

In view of the existence of quantum engagement even in the absence of any overlap between the spatial wavepackets we ask whether it could persist into the realm of classical physics. The answer is a resounding no by examining $|1, 1'\rangle$ of equation (2). The fact that it is a pure state composed of a linear superposition of two components, $|1, 1\rangle$ and $|\alpha\rangle$, embodies the quintessence of quantum mechanics.

It might seem that the physical implications of equations (10)–(12) could carry over to fermions, for the role of statistics in just these equations themselves is not so transparent. That this is not true is reflected in the allowed values of P_1 and P_1'. We recall that the internal state $|\phi_{\rm spin}\rangle$ of equation (5) for an atom in the mixing chamber is to be associated with every translational state $|\vec{k}\rangle$. Such a spin state makes sense, for any given $\vec{k}$, only when there are one or more such $\vec{k}$-atoms from source A and also from source B. Otherwise there could be some $\vec{k}$-atoms in $|1, 1\rangle$ dangling around without the company of some $\vec{k}$-atoms in $|1, 1'\rangle$. For fermions the only meaningful choice of occupation numbers is thus $n_{\vec{k}1} = 1$, $n_{\vec{k}1'} = 1$ or, equivalently, $P_1 = P_1' = \frac{1}{2}$. Consequently these two fixed probabilities P_1 and P_1' cannot serve as variables to change $|\psi_\pm\rangle$ and hence there is *no quantum engagement* for the case of free fermions.

6. Physical understanding of quantum engagement

To aid our physical understanding of the seemingly mysterious correlation we would like to link it to some other experiences that we are already well acquainted with. Let us return to equations (7)–(9). We reiterate that ρ_{coh} owes its existence entirely to the coherence of the phase relation between the amplitudes a and b of the components $|1, 1\rangle$ and $|\alpha\rangle$ in the state $|1, 1'\rangle$ of equation (2). Bringing on this coherence turns the unperturbed eigenstates states $|1, 1\rangle$ and $|\alpha\rangle$ of ρ_0 into the new pair of perturbed eigenstates $|\psi_\pm\rangle$ of the full ρ_{spin}. Hence each new pair can be viewed as the offspring of the coherent interference between the two unperturbed states. It is now instructive to compare with an analogous classical case of sound waves in a closed tube. Here the forward and the reflected sound waves in the closed tube are analogous to the unperturbed basis states $|1, 1\rangle$ and $|\alpha\rangle$. Each wave consists of the orderly vibrations superimposed on the random thermal motions of a macroscopic number of air molecules. However, if they were not allowed to interfere coherently, each wave would continue to exist without the influence of the other. Only the coherent interference of the two running waves within the closed tube results in the standing wave patterns (the eigenmodes). Similarly, if the two beams of bosons were not allowed to enter and mix in the common chamber, the two unperturbed states $|1, 1\rangle$ and $|\alpha\rangle$ would stay on forever. Only through their thorough mixing within the common chamber in a coherent manner would the new states $|\psi_\pm\rangle$ emerge; they are the analogues of the standing waves in the closed tube. No dynamical interactions among the particles are involved in either case.

7. Microscopic quantum interferences *en masse*

Since every momentum state $|\vec{k}\rangle$ is associated with the same internal state $|\psi_+\rangle$ or $|\psi_-\rangle$, once the atoms from sources A and B are well settled in the mixing chamber macroscopically large numbers $NP_\pm$ of them in various momenta would fall *en masse* into the same set of internal single-particle states $|\psi_\pm\rangle$, respectively. Analogously to the classical case, these are all a result of pure coherent interference without any dynamic interactions, except that they are of quantum origin this time. The distribution in the various momentum states $|\vec{k}\rangle$ depends as usual on temperature. Atomic thermal motions are thus present in both the classical and the quantal cases. But, upon averaging, these random motions do not affect the coherent, *macroscopic* outcome of the standing wave patterns in the former, nor that of the two macroscopic sets of magnetic moments $(NP_+)\boldsymbol{\mu}_+$ and $(NP_-)\boldsymbol{\mu}_-$ in the latter, $\boldsymbol{\mu}_\pm$ being the atomic magnetic moments in $|\psi_\pm\rangle$, respectively. Having established the analogy between the quantal and the classical cases we rephrase that, while $(P_+, |\psi_+\rangle)$ and $(P_-, |\psi_-\rangle)$ are the result of the *microscopic* quantum interference of the two states $|1, 1\rangle$ and $|\alpha\rangle$ in spin space that pertains to one single particle, the fact that every particle in the mixing chamber, regardless of its momentum, is at the receiving end of the *same* microscopic quantum interference effectively multiplies the microscopic result by N folds. This can thus be said to be a macroscopic number of quantum interferences occurring *en masse*, or together in a grand scale.

In comparison, the more familiar case of Josephson tunneling in the superconductivity version (or the Bose condensate version [17]) involves just a quantum interference with a definite phase difference between two macroscopic condensates of Cooper pairs (boson-like) on the two sides of a spatial junction, rather than a microscopic interference multiplied by N-fold. In other words, it is a macroscopic quantum interference in physical space rather than a multiple of simultaneous microscopic interferences in spin space. Furthermore, the Josephson tunneling is driven by a perturbative, dynamical tunneling Hamiltonian.

8. Experimental observation

Aside from the observation of the two stages of Bose–Einstein condensation into the $|\psi_+\rangle$ state and the $|\psi_-\rangle$ state, respectively [8], a measurement, for example, of the average dipole moment $\hat{\mu}_z$ in the mixed gas predicted to be

$$\langle N\hat{\mu}_z\rangle = N_+\langle\psi_+|\hat{\mu}_z|\psi_+\rangle + N_-\langle\psi_-|\hat{\mu}_z|\psi_-\rangle \tag{16}$$

would shed light on this case of microscopic quantum interferences *en masse*, while varying the ratio of $\frac{N_1}{N_1'}$ would illustrate the quantum engagement property of the $|\psi_+\rangle$ and $|\psi_-\rangle$ states.

9. Summary and conclusion

While the overlap of spatial wavefunctions or the *extrinsic* indistinguishability is known to bring about quantum statistical correlations in physical space, as manifested in Heisenberg exchange interaction and the BEC, the *intrinsic* indistinguishability together with the variability of boson populations (a signature of bosons) in the present case is shown to bring about a novel quantum statistical correlation in spin space. We call it ‘quantum engagement’ because of a parallel with the well known quantum entanglement. The effects of such quantum engagements and the *microscopic* quantum interferences occurring *en masse* can be seen in a coherently mixed Bose gas consisting of two kinds of atoms in different polarization states. Both the angle θ between the polarizations and the ratio of the populations of the two kinds of atoms serve as controlling parameters. It is not any dynamic potential but rather the *coherent interferences* in terms of these two parameters that bring about the novel quantum statistical correlation.

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References

[1] Ashcroft N W and Mermin N D 1976 *Solid State Physics* (New York: Holt, Rinehart and Winston)
[2] Einstein A 1924 *Sitzungsber. Preuss. Akad. Woss/(Berlin)* **22** 261
Einstein A 1925 *Sitzungsber. Preuss. Akad. Woss/(Berlin)* **23** 3
[3] Leggett A J 2001 *Rev. Mod. Phys.* **73** (2)
[4] Pethick C J and Smith H 2002 *Bose–Einstein Condensation in Dilute Gases* (Cambridge: Cambridge University Press)
[5] Dirac P A M 1926 *Proc. R. Soc.* **112** 661

[6] Jan J F and Lee Y C 1998 *Phys. Rev.* B **58** R1714
[7] Lee Y C and Zhu W 2000 *J. Phys.: Condens. Matter* **12** L49
[8] Lee Y C and Sablauer A 2005 *Phys. Rev. Lett.* **94** 130402
[9] Shlyapnikov G V *et al* 1994 *Phys. Rev. Lett.* **73** 3274
[10] Venturi V *et al* 1999 *Phys. Rev.* A **60** 4635
[11] Lyuboshitz V L and Podgorestskii M L 1971 *Sov. Phys.—JETP* **33** 5
Lyuboshitz V L and Podgorestskii M L 1972 *Sov. Phys.—Usp.* **14** 662
[12] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[13] Bell J S 1964 *Physics* **1** 195
[14] Bohm D 1989 *Quantum Theory* (New York: Dover)
[15] Anderson M H *et al* 1995 *Science* **269** 198
[16] Davis K B 1995 *Phys. Rev. Lett.* **75** 3969
[17] Marino I *et al* 1999 *Phys. Rev.* A **60** 487
Williams J E 2001 *Phys. Rev.* A **64** 013610